Amplified spontaneous emission I. The threshold condition

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Amplified spontaneous emission
I. The threshold condition

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Abstract. The threshold condition for the onset of amplified spontaneous emission is derived and experimentally verified in the 3·39 μm He–Ne and the 0·614 μm Ne systems. A short discussion is given concerning the confusion between this type of radiation and superradiance.

1. Introduction

In a recent paper (Allen and Peters 1970) we discussed the semantic confusion surrounding the word superradiance. The highly directional intense radiation emitted by an extended medium with a randomly prepared population inversion, in the absence of a laser cavity, was shown to be a manifestation of amplified spontaneous emission (ASE) rather than real superradiance as described by Dicke (1954). The approximate threshold condition for ASE to occur was derived, and the critical inversion required was related in a single expression to the length of the tube containing the excited atoms and to a few atomic parameters. The condition was verified by a single original measurement and by re-interpreting the published results of other workers.

It is surprising, perhaps, that although this type of radiation is now well known, no analysis had previously appeared which derived the critical conditions for these effects to occur, except that of Carver (1966) which confused the issue with enhancement factors supposedly due to Dicke-type superradiance.

In this work an alternative and simple derivation of the threshold condition is presented and the parametric relationships tested by detailed experiments on the 3·39 μm He–Ne and the 0·614 μm Ne transitions. The same systems have been investigated in detail to evaluate the temporal and spatial coherence of the light, beam divergence, polarization and intensity as a function of inversion density, tube length and bore. The theory of these effects and the results and their interpretation will be discussed in subsequent papers, II–IV.

2. Theory

Consider an ensemble of atoms of population inversion density n in a column of length L and cross-sectional area a. If N photons pass through the tube cross section per second at any point, then the net number of atoms induced to emit per unit volume per second at that point may be written $N\sigma n/a$, where σ is the resonance absorption cross section. Clearly $\sigma N/a$ is equivalent to $B\rho(\nu)$, where B is the Einstein coefficient and $\rho(\nu)$ the radiation density. When B is written in terms of τ_{21}, the lifetime of the transition, and $\rho(\nu)$ is replaced by $Nh\nu/ac\Delta\nu_{\mathrm{D}}$, where $\Delta\nu_{\mathrm{D}}$ is the width of the Doppler broadened transition, we find

$$\sigma = \frac{c^2}{8\pi\nu^2}\frac{1}{\Delta\nu_{\mathrm{D}}}\frac{1}{\tau_{21}}.$$

The appropriate rate equation for the change of number of photons δN by stimulated emission in the volume La in time δt is

$$\delta N = Ln\sigma N\delta t.$$

The threshold condition for ASE is realized when a spontaneously emitted photon at one end of the column just induces another at the other end. If there is one spontaneously emitted photon present then $N\delta t = 1$ and for one photon to be induced $\delta N = 1$. This yields the critical condition

$$n_c = \frac{8\pi\Delta\nu_D\tau_2}{L\lambda^2\phi}$$

where ϕ is the branching ratio and τ_2 is the lifetime of the excited state. If τ_2 were expressed in terms of a linewidth then, except for a factor of π, this is precisely the same condition as derived previously by the authors by considering the excited atoms to spontaneously radiate isotropically into all available modes. The agreement in the two approaches comes about obviously because for one photon the rate of stimulated emission into one mode equals the rate of spontaneous emission; thus in both derivations the condition for the onset of stimulated emission is being derived. This derivation, however, accounts properly for the effect of the finite population of the lower level of the transition, and for $\Delta\nu_D$, τ_2 and ϕ which were essentially phenomenologically introduced in the previous derivation. The calculation is, of course, simply a thinly disguised Schawlow and Townes (1958) laser threshold condition in the absence of a cavity. The condition may, alternatively, be written as

$$L_c = \frac{8\pi\Delta\nu_D\tau_2}{n\lambda^2\phi}$$

for an inversion density n.

3. Comparison of theory with experiment

Two systems have been experimentally investigated and these involve the transition at 3·39 μm He–Ne, which produces a continuous wave output, and the 0·614 μm line in pure Ne which only works in a pulsed régime. The He–Ne was rf excited and plasma tubes of length 310 cm, with bores varying between 2 and 4 mm in 0·5 mm stages, were used. The ASE intensity was measured as a function of length by disconnecting the rf clips on the far end of the tube away from the detector so that the detector geometry remained constant. The measurements were carried out for various levels of excitation, care being taken to ensure that the inversion was constant in all parts of the tube by monitoring the spontaneous emission from the upper level as seen at 0·633 μm. The spontaneous emission could be controlled to an accuracy of 5%. Figure 1 shows intensity against tube length for a number of inversion densities. It should be noted that for a given tube bore the cut-off length L_c varies with the inversion n. Figure 2 shows $1/L_c$ plotted against inversion, the result in agreement with theory is a straight line through the origin. The slope of the curve shows, assuming the appropriate values for ϕ, λ, $\Delta\nu_N$ (Faust and McFarlane 1964), that the inversions occurring were of the order of 10^9 atoms/cm^3. This was in agreement with the values obtained by investigating the output intensity as a function of length for tube lengths just above L_c. In this régime the intensity, as will be discussed in the next paper, grows exponentially with a coefficient simply related to the inversion and to the critical length. Table 1 shows the exact comparison.

Table 1. Inversion density in He–Ne

n from critical inversion relationship (10^8 atoms/cm^3)	n from gain coefficient measurement (10^8 atoms/cm^3)
1·93	5·55
1·67	4·82
1·46	4·30
1·23	3·59

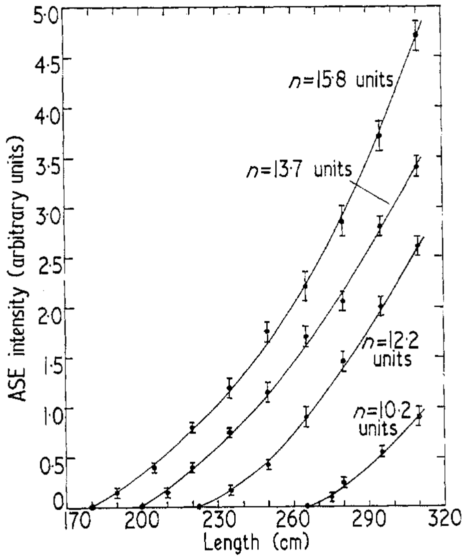

Figure 1. Plot of ASE intensity against length for a range of inversion densities in He–Ne.

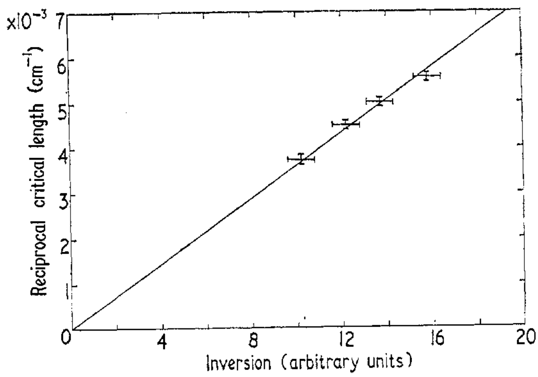

Figure 2. Plot of $1/L_c$ against inversion density for He–Ne.

The numbers are further substantiated by interpretation of the spontaneous emission from the energy levels involved. Examination of the lifetimes, degeneracies and line intensities concerned show that the lower level will have less than 1% of the population of the upper level. Consequently the inversion will be almost exactly proportional to the intensity of any transition from the upper level. The slope of critical length against spontaneous emission at 0·633 μm, as measured through the side of the tube, again yielded inversion values of the order of 10^9 atoms/cm^3. The standard deviation in the values of L_c for a given value of n for each of the different tube bores was less than 4%, verifying that the threshold condition is independent of bore and that the $d^2 \leqslant \lambda L$ criterion discussed by Carver (1966) is not meaningful.

The pure neon system was pulse excited, and fourteen tubes with lengths between 44 and 110 cm and a bore of 2·5 mm were investigated. The Ne was excited using a similar system to that of Geller *et al.* (1966). Fields of 260 to 480 Vcm^{-1} yielded 25 ns rise-time current pulses with peak amplitudes of up to 60 A. The optimum neon pressure was found to be 2·3 torr—rather higher than that used by other workers. The gain of the system was maintained at a constant level for a chosen field by arranging that the distance between the electrodes was kept constant even though the length of gas contributing to ASE was varied. The output was observed only from the live end of the tube in contrast to the observations of Clunie *et al.* (1965). Similar results for ASE intensity against length, and for $1/L_c$ against population inversion density,

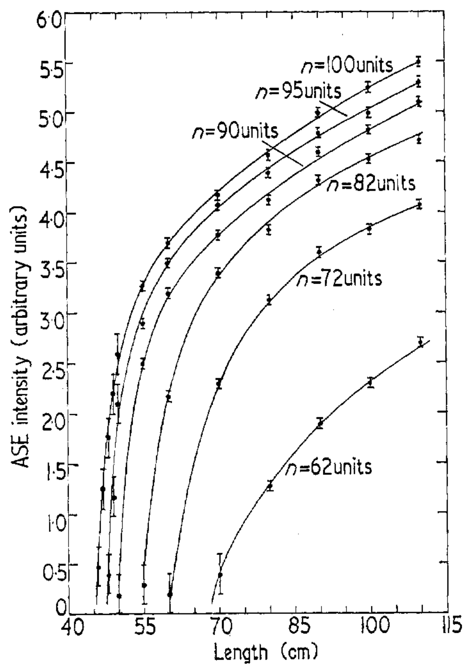

Figure 3. Plot of ASE intensity against length for a range of inversion densities in Ne.

are shown in figures 3 and 4. No check was made of the dependence of L_c upon bore but in this system L_c varies between 45 and 70 cm for the excitation levels used, whereas the $d^2 \leqslant L\lambda$ criterion predicts a minimum length of the order of 1000 cm!

The inversion density could not be deduced from the intensity curves for this system because the exponential region is very small. It would be desirable to make an accurate measurement of the inversion density but the difficulties associated with a pulsed system are such that it is not easy. However, again at the peak of the pulse the excitation cross sections are such (see Isaev and Petrash 1969) that the upper level may be expected to be approximately 20 times as populated as the lower. Thus monitoring the peak spontaneous emission intensity may be expected to give a reasonably satisfactory result. Figure 4 shows the resulting curve and table 2 tabulates the

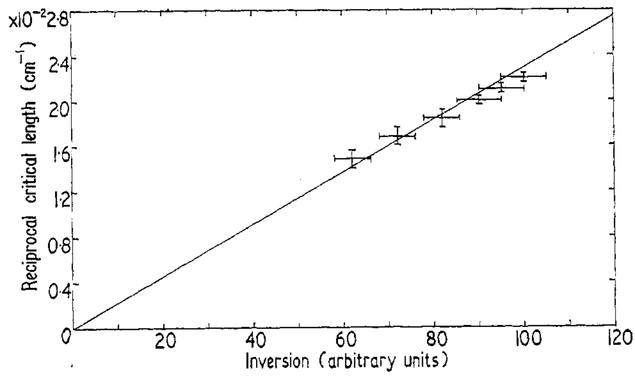

Figure 4. Plot of $1/L_c$ against inversion density for Ne.

Table 2. Inversion density in Ne

Electric field along discharge tube ($V\ cm^{-1}$)	n from critical inversion relationship (10^{10} atoms/cm^3)
480	1·16
420	1·10
365	1·04
330	0·95
295	0·83
260	0·72

inversion densities measured from the values of L_c using the values of ϕ and τ_2 given by Shoffstall and Ellis (1970), and assuming that $\Delta\nu_D$ is that of the 0·540 μm Ne line modified by the ratio of the frequencies (Egorov and Plekhotkin 1969).

In each system the critical length L_c was taken as that value at which the detected signal began to grow above the spontaneous emission emitted from the end of the tube. The spontaneous emission background was approximately constant for all lengths shorter than L_c for a given inversion density in the Ne system, and was undetectable in the case of He–Ne.

4. Conclusions

The simple relation between critical length for the onset of ASE and inversion derived in this paper has been verified in two different types of system. This vindicates the view that the use of the word superradiance, which many workers use to describe the radiation from such systems, is indeed a semantic confusion. This view is supported by the arguments concerning linewidth made in our earlier paper (Allen and Peters 1970).

It would appear that if a high population inversion could be achieved by a burst of electrons in a time very short compared with the lifetime of the upper level, then real superradiance in Dicke's sense of the word could occur. In these circumstances, provided other relaxation processes and atomic motion did not destroy the effect, the atoms could find themselves in the radiation reaction field of the first atoms to emit spontaneously, and phase themselves appropriately for collective de-excitation. This might be expected to be the dominant process until the photon flux became so large that the effect of stimulated emission could again dominate, as it undoubtedly would in the case of a long column of atoms. This would then naturally explain the observed line narrowing, which would have nothing to do with the collective model proposed by Ernst and Stehle (1968). This effect, however, does not appear to be present in the ASE systems so far described in the literature, where the methods of excitation certainly do not satisfy the necessary conditions. It is interesting to note that in work on photodissociation lasers (see for instance Kasper and Pimentel 1964), where conceivably a near perfect inversion is created very quickly at all points along the tube, the emission begins with a narrow fast rise-time peak. Arguably, this could be a collective spontaneous emission effect but it is difficult to establish this firmly.

There is one other possible effect which could be looked for, which is that, after a certain path length, the field could in principle be large enough to act as a θ pulse (see Dicke 1954). In this case superradiance and self-induced transparency effects (McCall and Hahn 1969) could begin to be important. This would be impossible in a cw system because of the dephasing effects of continual collisions, and unlikely in a pulsed system because of the upper limit implied to the length of the excited atom column by the need to excite all atoms simultaneously. The condition that the atoms be excited simultaneously is, of course, the same as exciting the atoms by a π pulse of coherent light. In each case all atoms would be in a well-defined state.

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